

Unfolding of Parametric Logical Regulatory Networks

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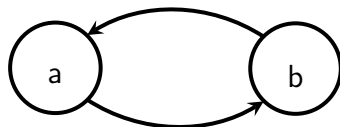
Motivation

1. Modelling of cellular processes.
2. Abstraction of precise kinetic parameters by discrete models.
3. Exploration of dynamics of all possible models.

Our Contribution

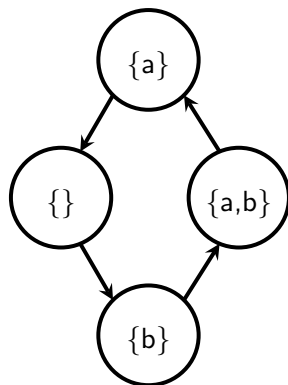
1. Use of parametric logical regulatory networks.
2. Novel encoding of the parameter space.
3. Partial order reduction of the traces.

Boolean Networks

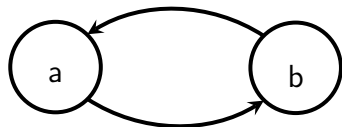


b	a
0	0
1	1

a	b
0	1
1	0

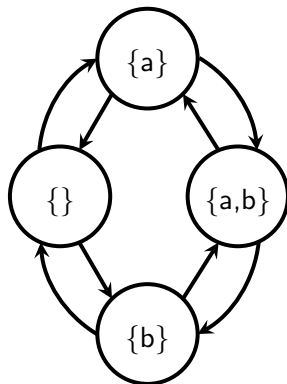


Parametric Boolean Networks



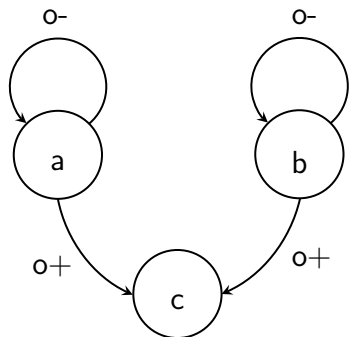
b	a
0	P_{\emptyset}^a
1	$P_{\{b\}}^a$

a	b
0	P_{\emptyset}^b
1	$P_{\{a\}}^b$



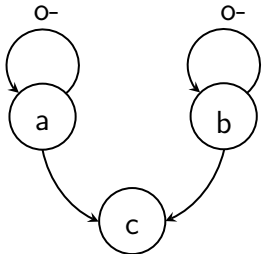
- ▶ The Boolean functions are inferred from biological data – which is often incomplete.
- ▶ We abstract the unknown in the form of parameters.

Edge Labels

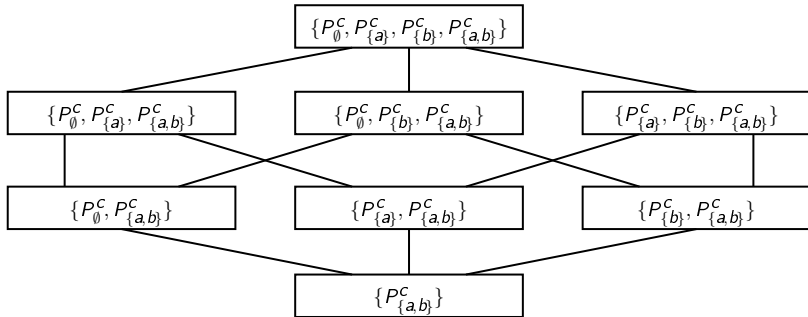


a	b	c
0	0	0
0	1	$P_{\{b\}}^c$
1	0	$P_{\{a\}}^c$
1	1	1

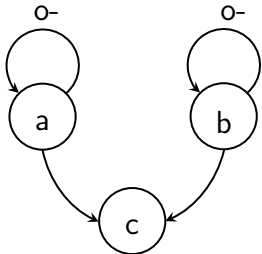
Parametrisation Encoding



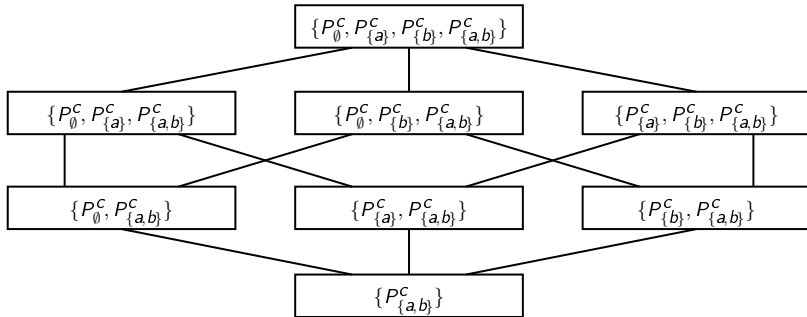
a	b	c
0	0	P_{\emptyset}^c
0	1	$P_{\{b\}}^c$
1	0	$P_{\{a\}}^c$
1	1	$P_{\{a,b\}}^c = 1$



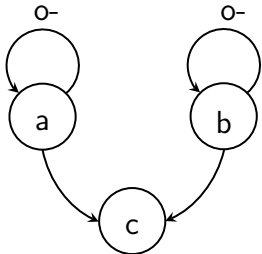
Parametrisation Encoding



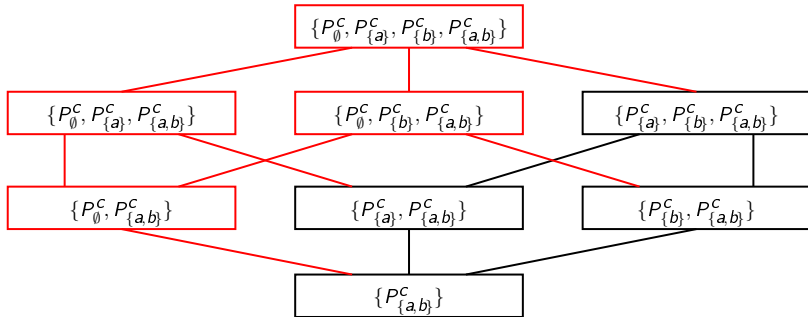
a	b	c
0	0	$P_{\emptyset}^c = 0$
0	1	$P_{\{b\}}^c$
1	0	$P_{\{a\}}^c$
1	1	$P_{\{a,b\}}^c = 1$



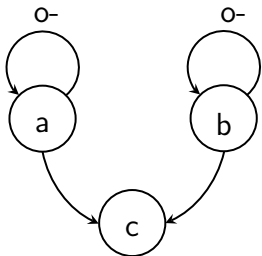
Parametrisation Encoding



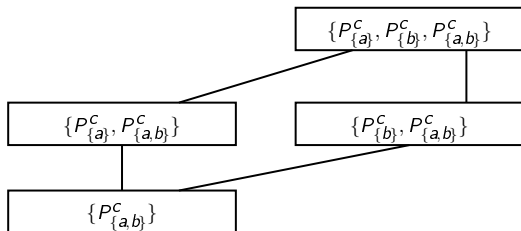
a	b	c
0	0	$P_{\emptyset}^c = 0$
0	1	$P_{\{b\}}^c$
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1	1	$P_{\{a,b\}}^c = 1$



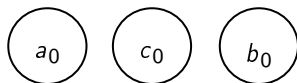
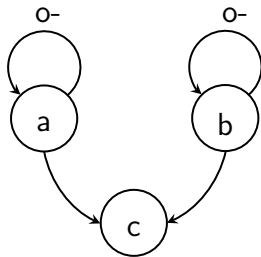
Parametrisation Encoding



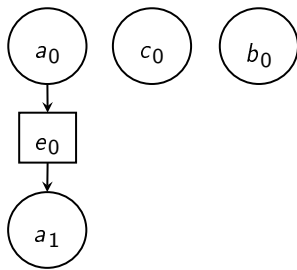
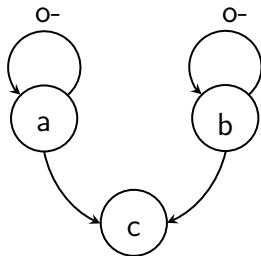
a	b	c
0	0	$P_{\emptyset}^c = 0$
0	1	$P_{\{b\}}^c$
1	0	$P_{\{a\}}^c$
1	1	$P_{\{a,b\}}^c = 1$



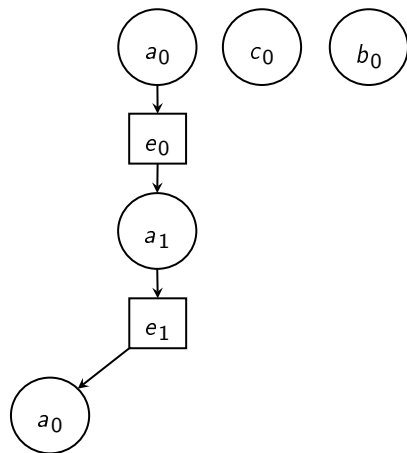
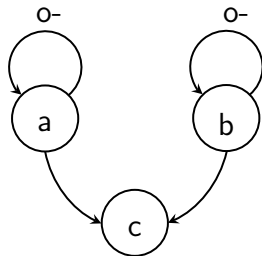
Unfoldings



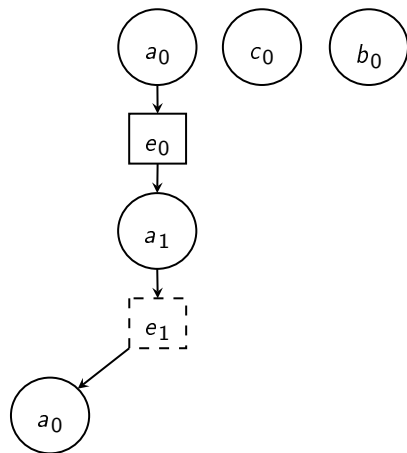
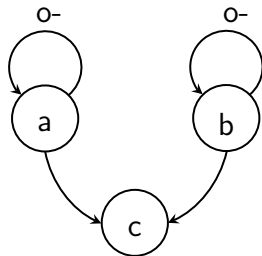
Unfoldings



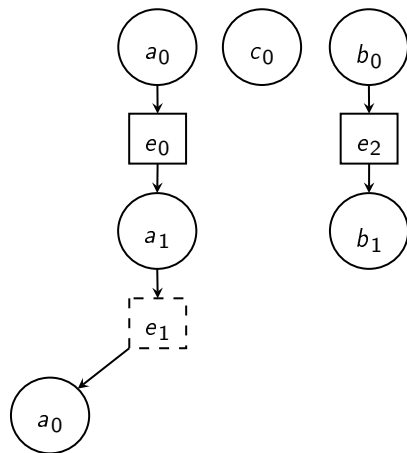
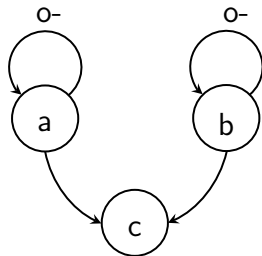
Unfoldings



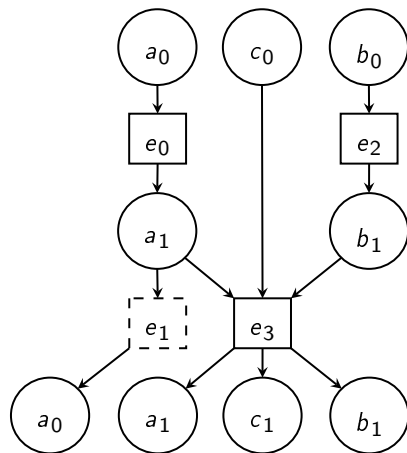
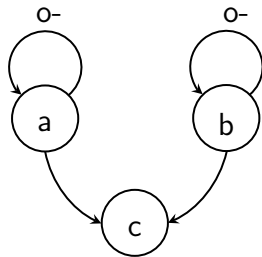
Unfoldings



Unfoldings



Unfoldings



Parametric Unfoldings

- ▶ Cut-off events only if no new parametrisation is introduced.
- ▶ No reasonable upper bound can be given on the size of the unfoldings.

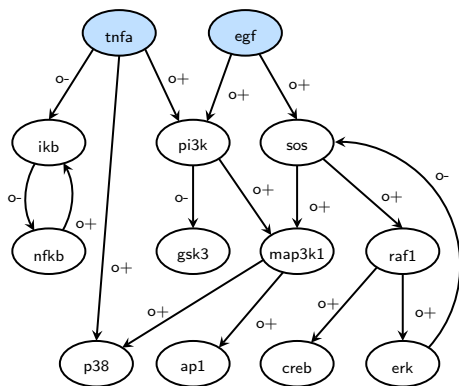
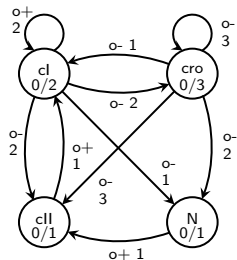
Experiments

- ▶ Experiments were conducted using a prototype application for parametric unfoldings¹
- ▶ We compare against state-of-the-art methods in parametric logical network analysis².

¹<https://github.com/GeorgeKolcak/Pawn>

²Gallet et al., ICFEM 2014

Experiments



Model	Unfolding Events	Symbolic Execution Size
Lambda Switch	170 (575)	68,011
LS (with Min/Max)	170 (569)	15,139
EGF-TNF α	1,057 (2,658)	534,498

Summary and Future Work

- ▶ We provide a platform for efficient parametric logical regulatory networks analysis.
- ▶ New encoding of parametrisations is introduced to battle combinatorial explosion.
- ▶ Partial order reduction using Petri net inspired unfoldings for parametric networks.
- ▶ Experimental results show great potential for the method in sparse networks.

Future Work

- ▶ Improve on parallelism to allow more concurrency.
- ▶ Efficient model checking using the overapproximation.