Synthesis of Petri Nets with Whole-place Operations and Localities

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Outline

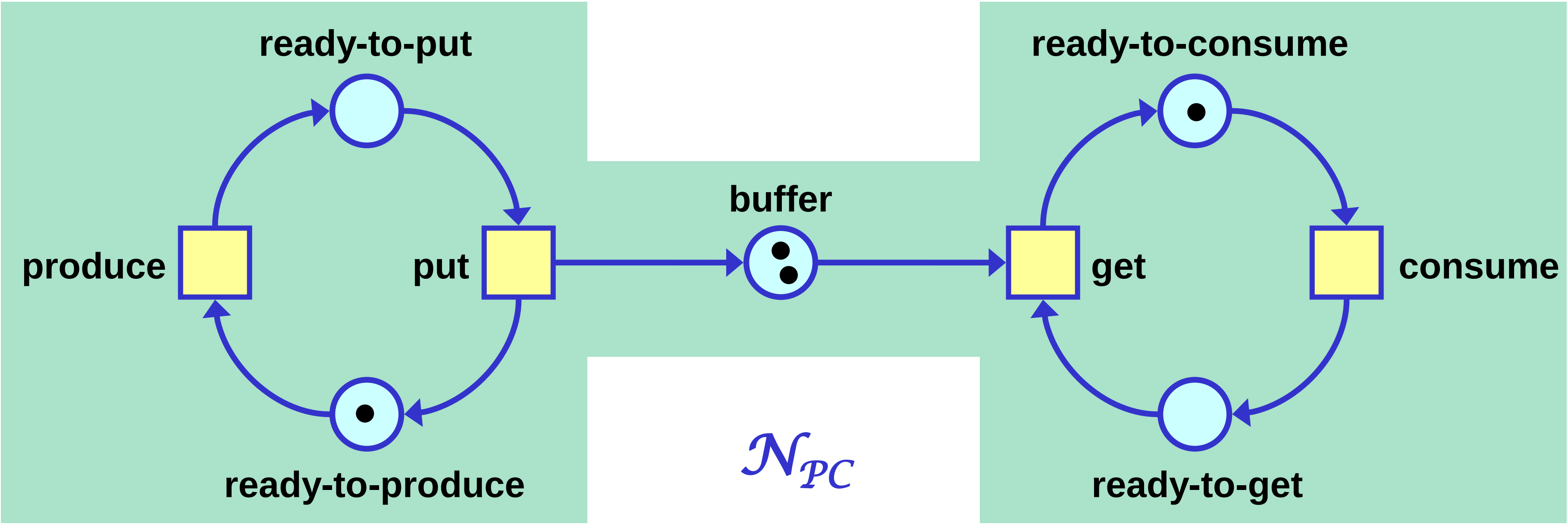
- motivation
- Petri nets
- synthesising Petri nets from step transition systems
  - synthesis problem
  - net-types and regions
  - net-type for Petri Nets with Whole-place Operations and Localities (WPOL-nets)
  - solving synthesis problems (feasibility and effective construction)
- future work

Synthesis of systems: motivation

- automated synthesis from behavioural specifications: attractive rigorous approach of constructing computational systems
- guarantees that the resulting systems are correct by design, hence time consuming verification is not needed
- HERE: systems are represented by Petri nets
- HERE: specifications are labelled transition systems
- one can also synthesise Petri nets from languages
- fast growing area of research related to synthesis: “mining” processes from logs of observed behaviour

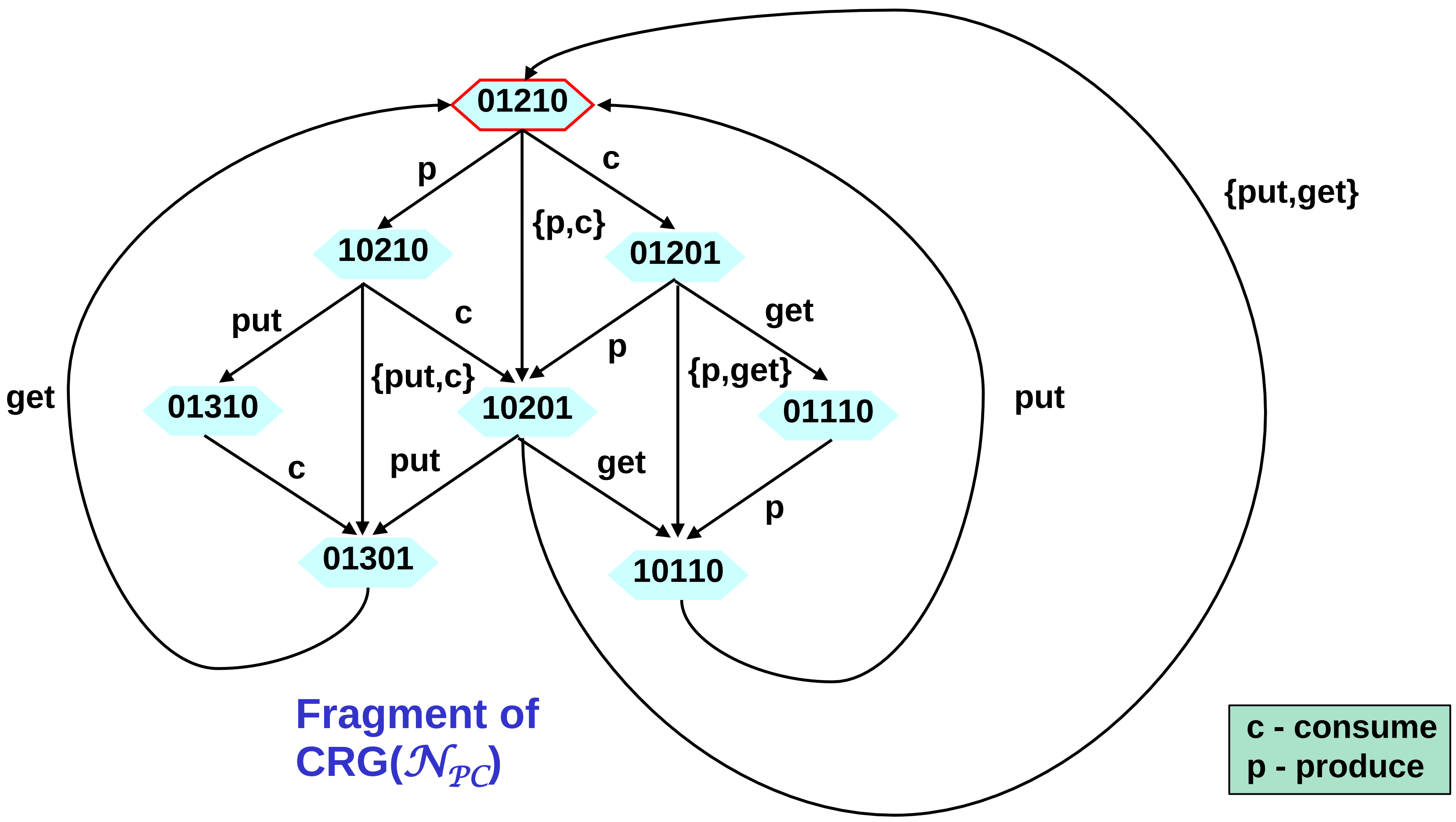
Modelling systems with Petri nets

- many kinds of Petri nets with different expressive power
- example: producer/consumer (modelled using a class of Ordinary Petri Nets)
  - producer repeatedly produces 1 item
  - consumer repeatedly consumes 1 item
  - unbounded buffer



Executing a Petri net

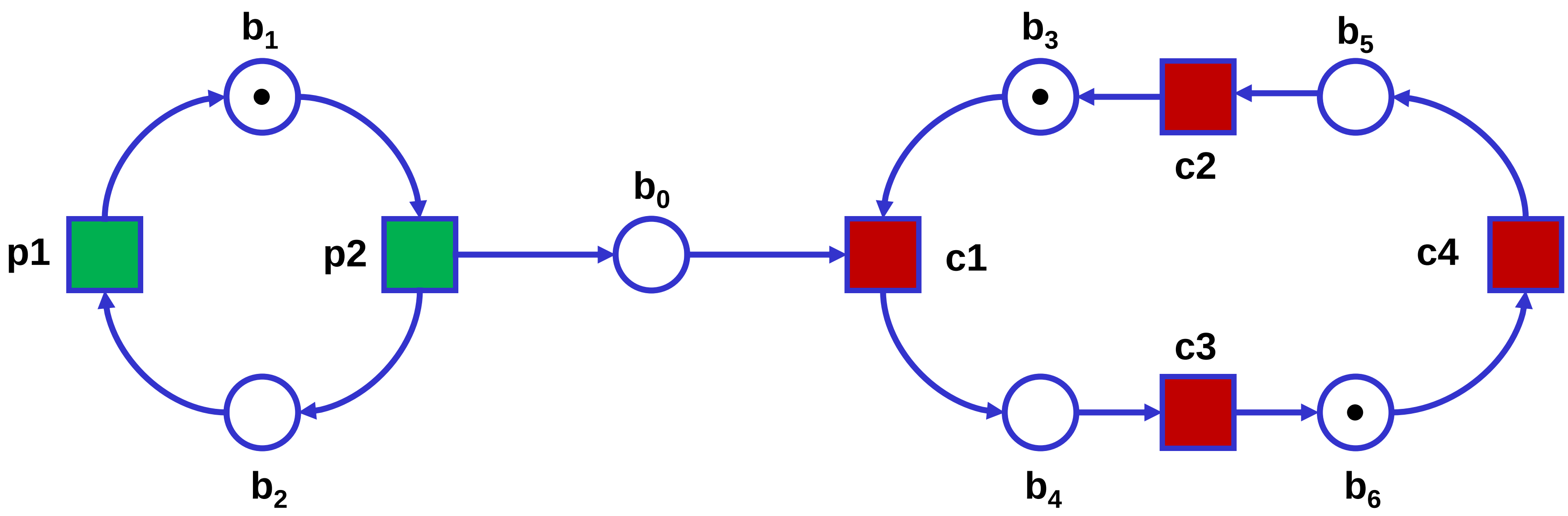
by firing steps (multisets of transitions) we can move from one marking to another generating the concurrent reachability graph (CRG) of a net



Fragment of CRG($\mathcal{N}_{\mathcal{PC}}$)

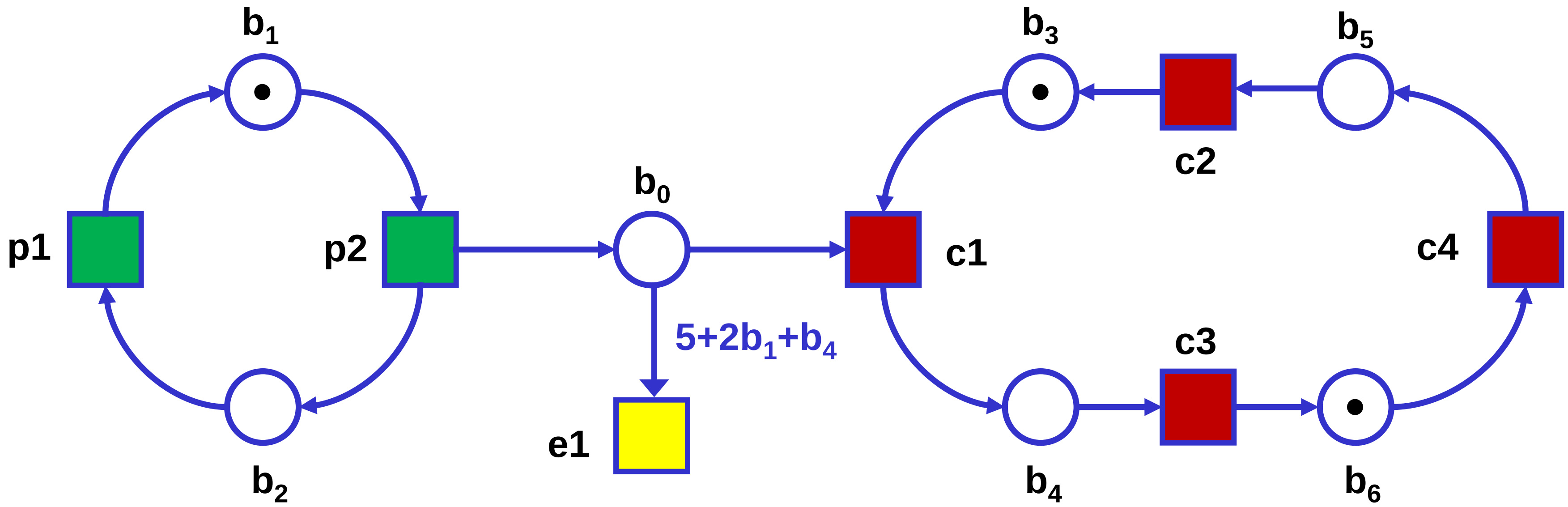
Petri nets with localities

- modified example: one producer and two consumers
- consumers are co-located
- locally maximal execution semantics (step firing policy)
- step sequence {p2}{c1} is illegal
- {p2}{c1, c4} and {p2}{c1, c4, p1} are legal



Petri nets with whole-place operations and localities

- Ordinary Petri Nets: unannotated arcs carry one token per firing
- PT-nets: arcs annotated with $n \geq 2$ carry n tokens per firing
- Petri Nets with Whole-place Operations: arcs annotated with linear expression involving places carry (per firing) variable numbers of tokens determined by the current marking



Application of nets with localities

- nets with localities can be used to describe and analyse globally asynchronous locally (maximally) synchronous systems (GALS)

- examples:
  - VLSI chips with multiple clocks for synchronisation of different subsets of gates
  - membrane systems modelling cells inside which reactions are carried out in co-ordinated pulses

Application of nets with whole-place operations

nets with whole-place operations can model:

- message passing interface (MPI) programs that perform parallel computations in the environment of distributed memory

- biological systems where the activation of reactions depends on the relative concentration of specific molecules and catalysts

Step transition systems

- behavioural model for WPOL-nets

- step transition system $TS = \langle Q, \mathbb{N}^T, \delta, q_0 \rangle$:
  - Q states
  - $q_0 \in Q$ initial state
  - $\delta: Q \times \mathbb{N}^T \rightarrow Q$ partial function describing arcs labelled by multi-sets of (net) transitions from T executed concurrently
- assumption: every state $q \in Q$ is reachable from q_0

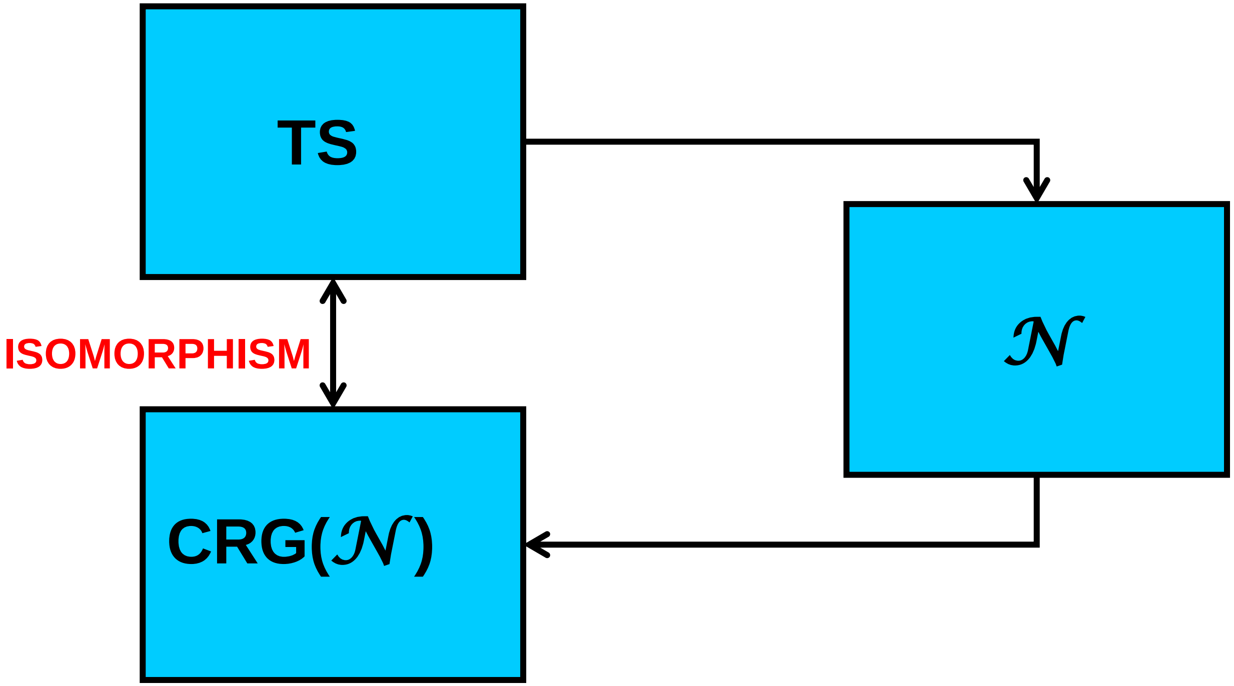
- TS is bounded if there is finite number of outgoing arcs at its every state
- TS is finite if it is bounded and has finitely many states

General synthesis problem

TS given step transition system

$\mathcal{N}$ Petri net obtained from TS through synthesis procedure

Aim: isomorphism between CRG($\mathcal{N}$) and TS



Regions and net-types

- to solve a synthesis problem one needs to construct enough net places using the information contained in TS
- suitable places are represented by regions, which depend on parameter τ (net-type)
- $\tau = \langle Q, S, \Delta \rangle$ is an uninitialized transition system over an abelian monoid S capturing the behaviour of a place of a net of a particular type
  - Q - specifies the values that can be stored in a place
  - S - specifies the nature of connections between a place and net transitions
  - Δ - gives the enabling conditions and the newly generated values for steps of transitions
- net-type for PT-nets: $\tau_{PT} = \langle \mathbb{N}, S_{PT}, \Delta_{PT} \rangle$, where $S_{PT} = \mathbb{N} \times \mathbb{N}$ and $\Delta_{PT}(n,(i,o)) = n - i + o$ provided that $n \geq i$

k-restricted WPOL-nets (k-WPOL-nets)

- problem: change of markings in WPOL-nets depends on current markings
- instead of constructing individual places we need to construct clusters of related places
- two places are related if (at least) one of them is a whole-place used in the annotations of arcs adjacent to the other place
- we partition places of a net into clusters (of no more than k places), with no exchange of whole-place marking information between different clusters
- k-WPOL-nets: WPOL-nets with a set of places partitioned into clusters of no more than k related places
- every k-WPOL-net can be expressed as a WPOL-net

Net-type for k-WPOL-nets

- a class of nets can be expressed as a class of τ-nets if we find a suitable net-type τ to describe its places
- k-WPOL-nets are not τ-nets according to the original definition
- extended net-type capturing the behaviour of sets of k places:

$$\tau^k = \langle \mathbb{N}^k, (\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k, \Delta^k \rangle, \text{ where}$$

matrices specifying the nature of connection between a cluster of places and a transition

$$\Delta^k : \mathbb{N}^k \times ((\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k) \rightarrow \mathbb{N}^k$$

vectors specifying the number of tokens in a cluster of k places

Synthesis problems

Problem 1 (feasibility)

TS is a bounded step transition system, $k > 0$, and ℓ is a locality mapping for transitions in TS

Provide necessary and sufficient conditions for TS to be realised by some τ^k -net $\mathcal{N}$ executed under the locally maximal step firing policy defined by ℓ, i.e. $TS \cong CRG_{\ell}(\mathcal{N})$

Problem 2 (effective construction)

TS is a finite step transition system, $k > 0$, and ℓ is a locality mapping for transitions in TS

Decide whether there is a finite τ^k -net, realizing TS when executed under the locally maximal step firing policy defined by ℓ. If the answer is positive construct such a τ^k -net

τ^k-regions

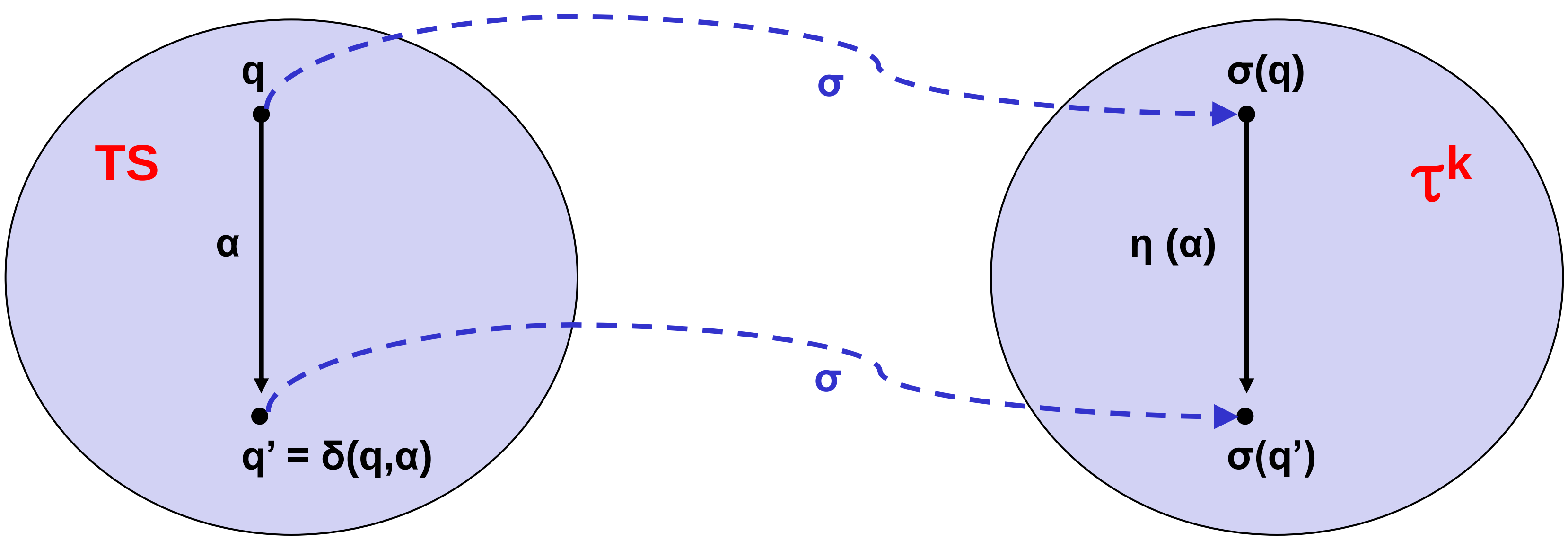
solving Problem 1: define a τ^k-region of $TS = \langle Q, \mathbb{N}^T, \delta, q_0 \rangle$ as two mappings $\langle \sigma, \eta \rangle$:

$$\sigma : Q \rightarrow \mathbb{N}^k$$
$$\eta : T \rightarrow (\mathbb{N}^{k+1})^k \times (\mathbb{N}^{k+1})^k$$

such that, for every state $q \in Q$ and every step α enabled at q in TS,

$$\eta(\alpha) = \sum_{t \in T} \alpha(t) \cdot \eta(t)$$

is enabled at $\sigma(q)$ in τ^k and $\Delta^k(\sigma(q), \eta(\alpha)) = \sigma(\delta(q,\alpha))$



Separation axioms

- definition of a region guarantees that regions of TS allow no less behaviour than TS, but they can allow more!
- to construct a net realising TS, we must find enough regions, so that the following hold:
  - Axiom 1 (state separation): for any states $q \neq r$ of TS, there is a τ^k-region $\langle\sigma, \eta\rangle$ of TS that distinguishes these states ($\sigma(q) \neq \sigma(r)$)
  - Axiom 2 (forward closure): for every step α that is not enabled in TS at some state q one of the following holds:
    - α is not region enabled: there is a τ^k-region $\langle\sigma, \eta\rangle$ of TS such that $\eta(\alpha)$ is not enabled at $\sigma(q)$ in τ^k
    - α is not control enabled: α is rejected by the locally maximal step firing policy

Main Result – solution to Problem 1

TS can be realised by a τ^k -net under the locally maximal step firing policy associated with ℓ

iff

Axioms 1 & 2 are satisfied

The challenge of solving Problem 2

- to solve Problem 2 using Main Result one needs to find an effective representation of τ^k-regions of TS
- we define system $\mathbf{S_{TS}}$ of equations and inequalities encoding the conditions to be satisfied by τ^k-regions
- the non-negative solutions of $\mathbf{S_{TS}}$ are in one-to-one correspondence with τ^k-regions of TS
- one can then check Axioms 1 & 2 for them
- for PT-nets, a similar procedure leads to a homogeneous linear system such that one can always find a finite basis for all the solutions (regions)
- for k-WPOL-nets, $\mathbf{S_{TS}}$ is (unfortunately) quadratic

Synthesis with known whole-places

- we consider sub-problem, Problem 3, assuming that we know all the whole-places of the net to be synthesised with their markings at every state of TS (no information about their connections to transitions yet!)
- as TS is finite, we have finite number of locality mappings to explore one-by-one, and so we can assume that ℓ is fixed
- knowing the markings of whole-places, we can deduce the ranges of the coefficients in the annotations of the arcs connecting these places with transitions from T, hence we can investigate them set-by-set, and so we can assume they are fixed
- every instance of a net $\mathcal{N}$ obtained as above (with only whole-places) can be checked whether it realizes TS

Adding non-whole-places

- if $\mathcal{N}$ does not realise TS we add non-whole-places to $\mathcal{N}$
- we build a modified system **S'_{TS}** to discover k-tuples of places with m whole-places and 1 generic non-whole-place (k = m + 1)
- **S'_{TS}** with many variables turned into concrete values due to the known information about whole-places is linear

vector specifying the coefficients in the annotation of the arc between non-whole-place p and some transition t

$$[\ m^q(p_1), \ldots, m^q(p_m), m^q(p), 1\ ] * [\ coeff_1, \ldots, coeff_m, 0, coeff_0\ ]$$

scalar product of two vectors, where red elements are variables and blue elements are values

Adding non-whole-places (cont.)

- each solution of **S'_{TS}** yields one non-whole-place
- one can always find a finite basis for all the solutions of **S'_{TS}**
- we can add the basis solutions (regions) to the set of regions corresponding to the whole-places and check Axioms 1 & 2
- if they are satisfied, the net obtained by adding extra non-whole-places to $\mathcal{N}$ is a solution to Problem 3; otherwise, there is no solution

Future work

- use the developed theory in selected case studies
- investigate the relationship between the locality mapping and the grouping of the places into clusters in k-WPOL-nets
- develop a synthesis approach for WPO-nets executed under more general step firing policies:
  - based on linear rewards of steps, where the reward for firing a single transition is fixed
  - based on linear rewards of steps, where the reward for firing a single transition depends on the current net marking

Thank you!